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We study light (current) quark masses in framework of chiral constituent quark model. In our calculation the current quark masses are defined uniquely, and all order effects of the light quark masses are considered. The results at energy scale $\mu = m_\rho$ are $m_s = (160 \pm 15)\text{MeV}$, $m_s/m_d = 20.2 \pm 3.0$, $m_u/m_d = 0.5 \pm 0.09$.

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I. INTRODUCTION

The quark masses are some of basic parameters of the standard model. There are various hadronic phenomenologies relating to the light quark (u , d , s) masses. For instance, they break the chiral symmetry of QCD explicitly. $m_u - m_d$ breaks isospin symmetry or charge symmetry, and $(m_u + m_d)/2 - m_s$ breaks SU(3) symmetry in hadron physics respectively. However, in QCD the masses of the light quarks are not directly measurable in inertial experiments, but enter the theory only indirectly as parameters in the fundamental lagrangian. The purpose of this paper is to study light quark masses at energy scale $\mu = m_\rho$ in a non-perturbative way.

In general, at low energy information about the light quark mass ratios is extracted in order by order by a rigorous, semiphenomenological method, chiral perturbative theory (ChPT) [1–3]. To the first order, the results, $m_s/m_d = 19$ and $m_u/m_d = 0.556$, are well-known [4]. Many authors have studied the mass ratios up to next to leading order of the chiral expansion. Gasser and Leutwyler first obtained $m_s/m_d = 20.2$ and $m_u/m_d = 0.554$ [3]. Then Kaplan and Manohar extracted $m_s/m_d = 15$ to 23 and $m_u/m_d = 0$ to 0.8 with very larger error bar [5]. These values have been improved to $m_s/m_d = (20.5 \pm 2.5)$, $m_u/m_d = 0.52 \pm 0.13$ by Leutwyler [6], to $m_s/m_d = 18$, $m_u/m_d = 0.66$ by Gerard [7], and to $m_s/m_d = 21$, $m_u/m_d = 0.30 \pm 0.07$ by Donoghue *et. al.* [8] respectively. Finally, Leutwyler analysis previous results and obtained $m_s/m_d = 18.9 \pm 0.8$, $m_u/m_d = 0.553 \pm 0.043$ [9]. So far, however, the study on light quark masses in framework of ChPT is limited by the following shortages: 1) In this framework, to obtain light quark mass ratio beyond the next to leading-order is very difficult, since more and more free parameters are included with raising of perturbative order. 2) Due to Kaplan-Manohar ambiguity [5], the definition of the light quark masses in ChPT is not uniquely. The reason has been pointed out in ref. [8] that, at the next to leading-order, QCD renormalization is mass-dependent, and the symmetry can not distinguish renormalized quark masses from “bare” quark masses. 3) In framework of ChPT, we can only obtain light quark mass ratios. For obtaining the individual quark masses, other approaches, such as QCD sum rules [10,11] or lattice calculation [12] are needed. These shortages are urgently wanted to be improved by theoretical studies of QCD and experiment.

In ref. [13] we have constructed chiral constituent quark model (ChCQM) including the lowest vector meson resonances following the spirit of Manohar-Georgi model [14]. This model provides a formulation to perform rigorous field theory calculation at energy scale lying between ChPT ($\mu \sim 0.5\text{GeV}$) and chiral symmetry spontaneously broken (CSSB) scale ($\mu \sim 1.2\text{GeV}$), and a successful description on physics in this energy region [13,15]. Up to $O(p^4)$, low energy limit of the model agree with ChPT well. Thus this model can be treated as an approach to extend ChPT investigation inspired by QCD. The most important advantage of this approach is that we can perform calculation beyond the low energy expansion in ChCQM, and only fewer free parameters are required. In general, there are three types of expansion working at low energy. They are momentum expansion, light quark mass expansion and N_c^{-1}

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expansion [16]. In ref. [13,15], we have provided a rigorous method to perform calculation to include all order terms of the momentum expansion and up to the next to leading order of N_c^{-1} expansion. In this the present paper, we will extend this method to reflect all order information of light quark mass expansion.

The Kaplan-Manohar ambiguity of ChPT has caused many debates. In particular, due to this ambiguity, the authors of ref [17] argued that the observed mass spectrum is consistent with a broad range of quark mass ratios, which specially includes the possibility $m_u = 0$. However, it is disagreed by other authors [18,8]. This problem can also be discussed in ChCQM. Since ChCQM is an effective approach with features of low energy QCD, light current quark masses are defined uniquely in ChCQM, that are just renormalized “physical” masses of u, d, and s quarks. In principle, therefore, there is no Kaplan-Manohar ambiguity in ChCQM. Light current quark masses can be determined uniquely via meson spectrum.

In ref. [15], we studied the isospin breaking process $\omega \rightarrow \pi^+\pi^-$ and determined isospin breaking parameter $m_d - m_u = (3.9 \pm 0.22)\text{MeV}$ at vector meson energy scale. This result together with meson spectrum provide so much information that we can obtain not only light quark mass ratios but also individual quark masses.

In ref. [13], we have shown that the chiral expansion at vector meson energy scale converge slowly. In particular, we have also pointed out that, if we neglect strange quark masses, the chiral expansion at $\phi(1020)$ energy scale will be divergent! It implies that m_s play very important role at ϕ -physics. In ref. [13,15] we have successfully studies the chiral expansion at m_ρ and m_ω energy scale. In order to extend this study to $K^*(892)$ and $\phi(1020)$, individual quark masses are necessary. It has been recognized that the light quark masses obtained in different approaches are with larger difference. Thus we have to extract quark masses by this formalism itself. It is one of our goals.

In general, in ChCQM the light quark masses can be extracted not only by pseudoscalar meson spectrums, but also by the lowest vector meson resonance spectrums. However, one-loop effects of mesons will contribute to vector meson masses, and calculation on one-loop effects of mesons is related to the chiral expansion at vector meson energy scale. As shown in ref. [13], this relation is very complicate. It makes that the relationship between vector meson spectrums and the light quark masses are also very complicate and indirect. Thus in this paper we will extract information about the light current quark masses from pseudoscalar meson spectrums and their decay constants. The vector meson spectrums will be predicted by this formalism in other paper.

The contents of the paper are organized as follows. In sect. 2 we review the basic notations of the chiral constituent quark model with the lowest vector meson resonances. In sect. 3, masses and decay constants of pseudoscalar meson octet are calculated. The results will including all order information of the light quark masses. In sect. 4, one-loop effects of pseudoscalar mesons and renormalization are discussed. The numerical results are given in sect. 5 and a brief summary is included in sect. 6.

II. CHIRAL CONSTITUENT QUARK MODEL

The simplest version of chiral quark model which was originated by Weinberg [1], and developed by Manohar and Georgi [14] provides a QCD-inspired description on the simple constituent quark model. In view of this model, in the energy region between the CSSB scale and the confinement scale ($\Lambda_{QCD} \sim 0.1 - 0.3\text{GeV}$), the dynamical field degrees of freedom are constituent quarks(quasi-particle of quarks), gluons and Goldstone bosons associated with CSSB(these Goldstone bosons correspond to lowest pseudoscalar octet). In this quasiparticle description, the effective coupling between gluon and quarks is small and the important interaction is the coupling between quarks and Goldstone bosons. In **I** we have further included the lowest vector meson resonances into this formalism. At chiral limit, this model is parameterized by the following chiral constituent quark lagrangian

$$\mathcal{L}_\chi = i\bar{q}(\not{\partial} + \not{V} + g_A \not{A}\gamma_5 - i\not{V})q - m\bar{q}q + \frac{F^2}{16} \langle \nabla_\mu U \nabla^\mu U^\dagger \rangle + \frac{1}{4}m_0^2 \langle V_\mu V^\mu \rangle. \quad (1)$$

Here $\langle \dots \rangle$ denotes trace in $SU(3)$ flavour space, $\bar{q} = (\bar{q}_u, \bar{q}_d, \bar{q}_s)$ are constituent quark fields. V_μ denotes vector meson octet and singlet. Since in this paper we only focus on pseudoscalar meson spectrums and decay constants, we will neglect vector meson fields in the following. The Δ_μ and Γ_μ are defined as follows,

$$\begin{aligned} \Delta_\mu &= \frac{1}{2} \{ \xi^\dagger (\partial_\mu - ir_\mu) \xi - \xi (\partial_\mu - il_\mu) \xi^\dagger \}, \\ \Gamma_\mu &= \frac{1}{2} \{ \xi^\dagger (\partial_\mu - ir_\mu) \xi + \xi (\partial_\mu - il_\mu) \xi^\dagger \}, \end{aligned} \quad (2)$$

and covariant derivative are defined as follows

$$\begin{aligned} \nabla_\mu U &= \partial_\mu U - ir_\mu U + iUl_\mu = 2\xi\Delta_\mu\xi, \\ \nabla_\mu U^\dagger &= \partial_\mu U^\dagger - il_\mu U^\dagger + iU^\dagger r_\mu = -2\xi^\dagger\Delta_\mu\xi^\dagger, \end{aligned} \quad (3)$$

where $l_\mu = v_\mu + a_\mu$ and $r_\mu = v_\mu - a_\mu$ are linear combinations of external vector field v_μ and axial-vector field a_μ , ξ associates with non-linear realization of spontaneously broken global chiral symmetry introduced by Weinberg [19]. This realization is obtained by specifying the action of global chiral group $G = SU(3)_L \times SU(3)_R$ on element $\xi(\Phi)$ of the coset space $G/SU(3)_V$:

$$\xi(\Phi) \rightarrow g_R \xi(\Phi) h^\dagger(\Phi) = h(\Phi) \xi(\Phi) g_L^\dagger, \quad g_L, g_R \in G, \quad h(\Phi) \in H = SU(3)_V. \quad (4)$$

Explicit form of $\xi(\Phi)$ is usual taken

$$\xi(\Phi) = \exp \{i\lambda^a \Phi^a(x)/2\}, \quad U(\Phi) = \xi^2(\Phi), \quad (5)$$

where the Goldstone boson Φ^a are treated as pseudoscalar meson octet. In ref. [13] we have shown that the lagrangian (1) is invariant under $G_{\text{global}} \times G_{\text{local}}$.

Distinguishing from some versions of chiral quark models, there is a kinetic term of pseudoscalar mesons in lagrangian (1). Therefore, the kinetic term of pseudoscalar mesons generated by one-loop effects of constituent quarks can be renormalized. Note that there is no mass term of pseudoscalar mesons in eq. (1). All of these are due to the basic assumption of the model [14]. In the other words, in this energy region, the dynamical field degrees of freedom are constituent quarks and massless Goldstone bosons (pseudoscalar octet) associated with CSSB. Masses of pseudoscalar mesons will be generated by quark loops as current quark mass parameters emerge in the dynamics (to see below). In ref. [13] we have fitted the parameter $g_A = 0.75$ via β -decay of neutron, and $m = 480\text{MeV}$ via low energy limit of the model. It has been also pointed out that the value of g_A has included effects of intermediate axial-vector meson resonances exchanges at low energy.

The light current quark mass-dependent term has been introduced in ref. [15] based on requirement of the chiral symmetry,

$$-\frac{1}{2}\bar{q}(\xi^\dagger \tilde{\chi} \xi^\dagger + \xi \tilde{\chi}^\dagger \xi)q - \frac{\kappa}{2}\bar{q}(\xi^\dagger \tilde{\chi} \xi^\dagger - \xi \tilde{\chi}^\dagger \xi)\gamma_5 q, \quad (6)$$

where $\tilde{\chi} = s + ip$, $s = s_{\text{ext}} + \mathcal{M}$, $\mathcal{M} = \text{diag}\{m_u, m_d, m_s\}$ is light current quark mass matrix, s_{ext} and p are scalar and pseudoscalar external field respectively. Eq. (6) will return to standard quark mass term of QCD, $\bar{\psi}\mathcal{M}\psi$, in absence of pseudoscalar mesons at high energy for arbitrary κ . Therefore, although the light current quark masses are defined uniquely in this formalism, the symmetry and the constraints of underlying QCD still can not fixed the coupling between pseudoscalar mesons and constituent quarks. κ will be treated as an initial parameter of the model and be fitted phenomenologically.

To conclude this section, the ChCQM lagrangian with light current quark masses is

$$\begin{aligned} \mathcal{L}_\chi = & i\bar{q}(\not{\partial} + \not{A} + g_A \not{A} \gamma_5)q - m\bar{q}q - \frac{1}{2}\bar{q}(\xi^\dagger \tilde{\chi} \xi^\dagger + \xi \tilde{\chi}^\dagger \xi)q - \frac{\kappa}{2}\bar{q}(\xi^\dagger \tilde{\chi} \xi^\dagger - \xi \tilde{\chi}^\dagger \xi)\gamma_5 q \\ & + \frac{F^2}{16} < \nabla_\mu U \nabla^\mu U^\dagger >, \end{aligned} \quad (7)$$

where vector meson fields have been omitted. The effects of isospin breaking due to inequality of light quark masses will be generated via constituent quark loops. For example, it generated masses of pseudoscalar mesons and splits their decay constants.

III. QUARK LOOPS

In this section, we will calculate pseudoscalar meson masses and decay constants induced by one-loop effects of constituent quarks. They are the leading order in N_c^{-1} expansion.

In this framework, the effective action describing meson interaction can be obtained via integrating over degrees of freedom of fermions

$$e^{iS_{\text{eff}}} \equiv \int \mathcal{D}\bar{q}\mathcal{D}q e^{i \int d^4x \mathcal{L}_\chi(x)} = < \text{vac}, \text{out} | \text{in}, \text{vac} >_{\text{Ext}}, \quad (8)$$

where $< \text{vac}, \text{out} | \text{in}, \text{vac} >_{\text{Ext}}$ is vacuum expectation value in presence external sources. The above path integral can be performed formally, and many methods, such as heat kernel manner [20,21], have been used to regulate the bilinear operator yielded via path integral. By using those formal integral methods, however, the explicit calculations of high order contributions to the chiral expansion are extremely tedious. In ref. [13] we have provided a convenient method

to evaluate effective action via calculating one-loop diagrams of constituent quarks directly. This method can capture all high order contributions of the chiral expansion.

In interaction picture, the equation(8) is rewritten as follow

$$\begin{aligned}
e^{iS_{\text{eff}}} &= \langle 0 | \mathcal{T}_q e^{i \int d^4x \mathcal{L}_\chi^I(x)} | 0 \rangle \\
&= \sum_{n=1}^{\infty} i \int d^4p_1 \frac{d^4p_2}{(2\pi)^4} \cdots \frac{d^4p_n}{(2\pi)^4} \tilde{\Pi}_n(p_1, \cdots, p_n) \delta^4(p_1 - p_2 - \cdots - p_n) \\
&\equiv i\Pi_1(0) + \sum_{n=2}^{\infty} i \int \frac{d^4p_1}{(2\pi)^4} \cdots \frac{d^4p_{n-1}}{(2\pi)^4} \Pi_n(p_1, \cdots, p_{n-1}),
\end{aligned} \tag{9}$$

where \mathcal{T}_q is time-order product of constituent quark fields, \mathcal{L}_χ^I is quark-meson interaction part of lagrangian(7), $\tilde{\Pi}_n(p_1, \cdots, p_n)$ is one-loop effects of constituent quarks with n external sources, p_1, p_2, \cdots, p_n are four-momentas of n external sources respectively and

$$\Pi_n(p_1, \cdots, p_{n-1}) = \int d^4p_n \tilde{\Pi}_n(p_1, \cdots, p_n) \delta^4(p_1 - p_2 - \cdots - p_n). \tag{10}$$

To get rid of all disconnected diagrams, we have

$$\begin{aligned}
S_{\text{eff}} &= \sum_{n=1}^{\infty} S_n, \\
S_1 &= \Pi_1(0), \\
S_n &= \int \frac{d^4p_1}{(2\pi)^4} \cdots \frac{d^4p_{n-1}}{(2\pi)^4} \Pi_n(p_1, \cdots, p_{n-1}), \quad (n \geq 2).
\end{aligned} \tag{11}$$

Hereafter we will call S_n as n -point effective action.

For the purpose of this paper, the lagrangian(7) can be rewritten as follow

$$\mathcal{L}_\chi(x) = \mathcal{L}_q(x) + \mathcal{L}_2^{(0)}(x) + \mathcal{L}[\pi(x), \eta_8(x)] + \mathcal{L}[K^\pm(x)] + \mathcal{L}[K^0(x)], \tag{12}$$

where \mathcal{L}_q and $\mathcal{L}_2^{(0)}$ are free field lagrangian of constituent quarks of pseudoscalar meson respectively,

$$\begin{aligned}
\mathcal{L}_q &= \sum_{i=u,d,s} \bar{q}_i (i/\partial - \bar{m}_i) q_i, \quad \bar{m}_i = m + m_i, \\
\mathcal{L}_2^{(0)} &= \frac{F^2}{16} \langle \nabla_\mu U \nabla^\mu U^\dagger \rangle \\
&= \frac{F^2}{8} \partial_\mu \Phi^a \partial^\mu \Phi^a + 4a_\mu^a \partial^\mu \Phi^a + \cdots, \quad a = 1, 2, \cdots, 8,
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
\lambda^a \Phi^a(x) &= \sqrt{2} \begin{pmatrix} \frac{\pi_3}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi_3}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 \end{pmatrix}, \\
\lambda^a a_\mu^a &= \sqrt{2} \begin{pmatrix} A_\mu^{(u)} & a_\mu^+ & A_\mu^+ \\ a_\mu^- & A_\mu^{(d)} & A_\mu^0 \\ A_\mu^- & \bar{A}_\mu^0 & A_\mu^{(s)} \end{pmatrix}.
\end{aligned}$$

$\mathcal{L}[\Phi(x)]$ denotes quark-meson coupling lagrangian,

$$\begin{aligned}
\mathcal{L}[\pi, \eta_8] &= \frac{g_A}{\sqrt{2}} [(\partial_\mu \pi^+ + 2a_\mu^+) \bar{u} \gamma^\mu \gamma_5 d + c.c.] + \frac{g_A}{2} \sum_{i=u,d,s} (\partial_\mu P_i + 2A_\mu^{(i)}) \bar{q}_i \gamma^\mu \gamma_5 q_i \\
&\quad + \frac{i}{\sqrt{2}} \kappa (m_u + m_d) (\pi^+ \bar{u} \gamma_5 d + c.c.) + i\kappa \sum_{i=u,d,s} m_i P_i \bar{q}_i \gamma_5 q_i
\end{aligned}$$

$$+\frac{1}{2}(m_u+m_d)\pi^+\pi^-(\bar{u}u+\bar{d}d)+\frac{1}{2}\sum_{i=u,d,s}m_iP_i^2\bar{q}_iq_i, \quad (14)$$

$$\begin{aligned} \mathcal{L}[K^\pm] &= \frac{g_A}{\sqrt{2}}[(\partial_\mu K^\pm + 2A_\mu^\pm)\bar{u}\gamma^\mu\gamma_5 s + c.c.] + \frac{i}{\sqrt{2}}\kappa(m_u+m_s)(K^\pm\bar{u}\gamma_5 s + c.c.) \\ &\quad + \frac{1}{2}(m_u+m_s)K^+K^-(\bar{u}u+\bar{s}s), \\ \mathcal{L}[K^0] &= \frac{g_A}{\sqrt{2}}[(\partial_\mu K^0 + 2A_\mu^0)\bar{d}\gamma^\mu\gamma_5 s + (\partial_\mu \bar{K}^0 + 2\bar{A}_\mu^0)d\gamma^\mu\gamma_5 \bar{s}] \\ &\quad + \frac{i}{\sqrt{2}}\kappa(m_d+m_s)(K^0\bar{d}\gamma_5 s + \bar{K}^0 d\gamma_5 \bar{s}) + \frac{1}{2}(m_d+m_s)K^0\bar{K}^0(\bar{d}d+\bar{s}s), \end{aligned}$$

where *c.c.* denotes charge conjugate term of pervious term,

$$P_u = \pi_3 + \frac{1}{\sqrt{3}}\eta_8, \quad P_d = -\pi_3 + \frac{1}{\sqrt{3}}\eta_8, \quad P_s = -\frac{2}{\sqrt{3}}\eta_8, \quad (15)$$

and $A_\mu^{(u)}$, $A_\mu^{(d)}$, $A_\mu^{(s)}$ etc. are axial-vector external fields correponding these pseudoscalar meson fields. From eq.(14) we can see that the K^\pm -quark coupling and K^0 -quark coupling are similar to π^\pm -quark coupling. Thus we only need to calculate masses and decay constants of π^\pm , π^0 and η^8 . Then masses and decay constants of K^\pm can be obtained via replacing m_d by m_s in one of π^\pm , and masses and decay constants of K^0 can be obtained via replacing m_u by m_s in one of π^\pm .

The one-point effective action is generated by tadpole-loop of constituent quarks. Calculation about this tadpole-loop contribution is simple.

$$\begin{aligned} iS_1[\pi, \eta_8] &= \frac{i}{2} \int d^4x \{ (m_u+m_d)\pi^+(x)\pi^-(x) < 0|T(\bar{u}(x)u(x) + \bar{d}(x)d(x))|0 > \\ &\quad + \sum_{i=u,d,s} m_i P_i^2(x) < 0|T\bar{q}_i(x)q_i(x)|0 > \\ &= -\frac{i}{2} \int d^4x \int \frac{d^4k}{(2\pi)^4} \{ [(m_u+m_d)\pi^+(x)\pi^-(x) + m_u P_1^2(x)] Tr[S_F^{(u)}(k)] \\ &\quad + [(m_u+m_d)\pi^+(x)\pi^-(x) + m_d P_2^2(x)] Tr[S_F^{(d)}(k)] + m_s P_3^2(x) Tr[S_F^{(s)}(k)] \}, \end{aligned} \quad (16)$$

where Tr denotes trace taking over color and Lorentz space, $S_F^{(q)}$ is propagator of constituent quark fields, $S_F^{(q)}(k) = i/(k - \bar{m}_q + i\epsilon)^{-1}$. In terms of dimensional regularization, we can integrate over internal line momenta k in the above equation. The result is

$$\begin{aligned} iS_1[\pi, \eta_8] &= -\frac{2N_c}{(4\pi)^{D/2}}\Gamma(1 - \frac{D}{2})i \int d^4x \{ (\frac{\mu^2}{\bar{m}_u^2})^{\epsilon/2}\bar{m}_u^3[(m_u+m_d)\pi^+\pi^- + m_u P_1^2] \\ &\quad + (\frac{\mu^2}{\bar{m}_d^2})^{\epsilon/2}\bar{m}_d^3[(m_u+m_d)\pi^+\pi^- + m_d P_2^2] + (\frac{\mu^2}{\bar{m}_s^2})^{\epsilon/2}\bar{m}_s^3 m_s P_3^2 \}. \end{aligned} \quad (17)$$

Defining a constant B_0 to absorbe the quadratic divergence from loop integral,

$$\frac{F_0^2}{16}B_0 = \frac{N_c}{(4\pi)^{D/2}}(\frac{\mu^2}{m^2})^{\epsilon/2}\Gamma(1 - \frac{D}{2})m^3, \quad (18)$$

we have

$$\begin{aligned} S_1[\pi, \eta_8] &= \int d^4x \mathcal{L}_1[\pi(x), \eta_8(x)] \\ \mathcal{L}_1[\pi, \eta_8] &= -\frac{F_0^2}{8}B_0\{(x_u^3+x_d^3)(m_u+m_d)\pi^+\pi^- + \sum_{i=u,d,s} x_i^3 m_i P_i^2\} \\ &\quad - \frac{N_c}{8\pi^2}m^3\{(x_u^3 \ln x_u^2 + x_d^3 \ln x_d^2)(m_u+m_d)\pi^+\pi^- + \sum_{i=u,d,s} m_i x_i^3 \ln x_i^2 P_i^2\} \end{aligned} \quad (19)$$

where $x_q = \bar{m}_q/m$.

The two-point effective action concerning to masses and decay constants of charge pion can be obtained as follow,

$$\begin{aligned}
iS_2[\pi^\pm] &= \frac{g_A^2}{2} \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} (iq_\mu \pi^+(q) + 2a_\mu^+(q)) (-iq_\nu \pi^-(q) + 2a_\nu^-(q)) Tr[\gamma^\mu \gamma_5 S_F^{(d)}(k-q) \gamma^\nu \gamma_5 S_F^{(u)}(k)] \\
&\quad + \frac{i}{2} g_A \kappa (m_u + m_d) \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \{ (iq_\mu \pi^+(q) + 2a_\mu^+(q)) \pi^-(q) Tr[\gamma^\mu \gamma_5 S_F^{(d)}(k-q) \gamma_5 S_F^{(u)}(k)] + c.c. \} \\
&\quad - \frac{\kappa^2}{2} (m_u + m_d)^2 \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \pi^+(q) \pi^-(q) Tr[\gamma_5 S_F^{(d)}(k-q) \gamma_5 S_F^{(u)}(k)] \\
&= \frac{2N_c}{(4\pi)^2} \Gamma(2 - \frac{D}{2}) g_A^2 i \int d^4 x \int \frac{d^4 q}{(2\pi)^4} e^{iq \cdot x} (iq_\mu \pi^+(q) + 2a_\mu^+(q)) (\partial^\mu \pi^-(x) + 2a^{-\mu}(x)) \\
&\quad \times \int_0^1 dt [\bar{m}_u^2 + \bar{m}_u \bar{m}_d + t(\bar{m}_d^2 - \bar{m}_u^2)] \left(\frac{\bar{m}_u^2 + t(\bar{m}_d^2 - \bar{m}_u^2) - t(1-t)q^2}{4\pi\mu^2} \right)^{-\frac{\epsilon}{2}} \\
&\quad + \frac{2N_c}{(4\pi)^2} \Gamma(2 - \frac{D}{2}) \kappa g_A (m_u + m_d) i \int d^4 x \int \frac{d^4 q}{(2\pi)^4} e^{iq \cdot x} [(iq_\mu \pi^+(q) + 2a_\mu^+(q)) \partial^\mu \pi^-(x) + c.c.] \\
&\quad \times \int_0^1 dt [\bar{m}_u + t(m_d - m_u)] \left(\frac{\bar{m}_u^2 + t(\bar{m}_d^2 - \bar{m}_u^2) - t(1-t)q^2}{4\pi\mu^2} \right)^{-\frac{\epsilon}{2}} \\
&\quad + \frac{2N_c}{(4\pi)^2} \kappa^2 (m_u + m_d)^2 i \int d^4 x \int \frac{d^4 q}{(2\pi)^4} e^{iq \cdot x} \pi^+(q) \pi^-(x) \int_0^1 dt \left(\frac{\bar{m}_u^2 + t(\bar{m}_d^2 - \bar{m}_u^2) - t(1-t)q^2}{4\pi\mu^2} \right)^{-\frac{\epsilon}{2}} \\
&\quad \times \{ \Gamma(1 - \frac{D}{2}) [\bar{m}_u^2 + t(\bar{m}_d^2 - \bar{m}_u^2) - t(1-t)q^2] - \Gamma(2 - \frac{D}{2}) [\bar{m}_u^2 - \bar{m}_u \bar{m}_d + t(\bar{m}_d^2 - \bar{m}_u^2) - 2t(1-t)q^2] \}.
\end{aligned} \tag{20}$$

There are both quadratic divergence and logarithmic divergence in the above effective action. The quadratic divergence can be canceled by constant B_0 defined in eq.(18), and the logarithmic divergence can be canceled via defining

$$\begin{aligned}
g^2 &= \frac{8N_c}{3(4\pi)^2} \Gamma(2 - \frac{D}{2}) \left(\frac{4\pi\mu^2}{m^2} \right)^{\frac{\epsilon}{2}}, \\
\frac{F_0^2}{16} &= \frac{F^2}{16} + \frac{N_c}{(4\pi)^2} g_A^2 m^2 \Gamma(2 - \frac{D}{2}) \left(\frac{4\pi\mu^2}{m^2} \right)^{\frac{\epsilon}{2}}.
\end{aligned} \tag{21}$$

g is an universal coupling constant of this model. In ref. [13], it has been determined as $g^2 = \frac{N_c}{3\pi^2}$ by the first KSFR sum rule [22].

Then eq.(20) together with eq.(19) give $O(N_c)$ effective lagrangian containing the terms linear or quadratic in the charge pion fields as follow

$$\begin{aligned}
\mathcal{L}_2[\pi^\pm(x)] &= \frac{F^2(m_u, m_d)}{4} \partial_\mu \pi^+ \partial^\mu \pi^- + \frac{\bar{f}^2(m_u, m_d)}{2} (a_\mu^+ \partial^\mu \pi^- + c.c.) - \frac{F_0^2}{4} \bar{M}^2(m_u, m_d) \pi^+ \pi^- \\
&\quad + \int \frac{d^4 q}{(2\pi)^4} e^{iq \cdot x} \left\{ \frac{\alpha(q^2; m_u, m_d)}{2} iq^\mu (a_\mu^+(x) \pi^-(q) + c.c.) - \frac{F_0^2}{4} \beta(q^2; m_u, m_d) \pi^+(q) \pi^-(x) \right\},
\end{aligned} \tag{22}$$

where

$$\begin{aligned}
F^2(m_u, m_d) &= F_0^2 + \frac{3}{2} g^2 g_A^2 (m_u + m_d) (4m + m_u + m_d) + 3g^2 \kappa g_A (m_u + m_d) (\bar{m}_u + \bar{m}_d) \\
&\quad - \frac{N_c}{2\pi^2} g_A [g_A (\bar{m}_u + \bar{m}_d) + 2\kappa (m_u + m_d)] \int_0^1 dt [\bar{m}_u + t(m_d - m_u)] \ln(x_u^2 + t(x_d^2 - x_u^2)) \\
&\quad + 4\kappa^2 (m_u + m_d)^2 \left\{ \left(\frac{g^2}{4} - \frac{F_0^2 B_0}{48m^3} - \frac{N_c}{24\pi^2} \right) - \frac{N_c}{8\pi^2} \int_0^1 dt \cdot t(1-t) [3 \ln(x_u^2 + t(x_d^2 - x_u^2)) - \frac{\bar{m}_u \bar{m}_d}{\bar{m}_u^2 + t(\bar{m}_d^2 - \bar{m}_u^2)}] \right\}, \\
\bar{f}^2(m_u, m_d) &= F_0^2 + \frac{3}{2} g^2 g_A^2 (m_u + m_d) (4m + m_u + m_d) + \frac{3}{2} g^2 \kappa g_A (m_u + m_d) (\bar{m}_u + \bar{m}_d) \\
&\quad - \frac{N_c}{2\pi^2} g_A [g_A (\bar{m}_u + \bar{m}_d) + \kappa (m_u + m_d)] \int_0^1 dt [\bar{m}_u + t(m_d - m_u)] \ln(x_u^2 + t(x_d^2 - x_u^2)),
\end{aligned}$$

$$\begin{aligned}
\bar{M}^2(m_u, m_d) &= B_0(m_u + m_d) \left\{ \frac{1}{2}(x_u^3 + x_d^3) + \frac{N_c}{2\pi^2} \frac{m^3}{F_0^2 B_0} (x_u^3 \ln x_u^2 + x_d^3 \ln x_d^3) - \frac{\kappa^2}{4} \frac{m_u + m_d}{m} (x_u^2 + x_d^2) \right\} \\
&+ \frac{3}{2} g^2 \kappa^2 \frac{(m_d^2 - m_u^2)^2}{F_0^2} - \frac{N_c}{2\pi^2} \kappa^2 \frac{(m_u + m_d)^2}{F_0^2} \int_0^1 dt [2\bar{m}_u^2 - \bar{m}_u \bar{m}_d + 2t(\bar{m}_d^2 - \bar{m}_u^2)] \ln(x_u^2 + t(x_d^2 - x_u^2)), \\
\alpha(q^2; m_u, m_d) &= -\frac{N_c}{2\pi^2} g_A [g_A(\bar{m}_u + \bar{m}_d) + \kappa(m_u + m_d)] \int_0^1 dt [\bar{m}_u + t(m_d - m_u)] \ln \left(1 - \frac{t(1-t)q^2}{\bar{m}_u^2 + t(\bar{m}_d^2 - \bar{m}_u^2)} \right), \\
\beta(q^2; m_u, m_d) &= \frac{N_c}{2\pi^2} g_A \frac{q^2}{F_0^2} [g_A(\bar{m}_u + \bar{m}_d) + 2\kappa(m_u + m_d)] \int_0^1 dt [\bar{m}_u + t(m_d - m_u)] \ln \left(1 - \frac{t(1-t)q^2}{\bar{m}_u^2 + t(\bar{m}_d^2 - \bar{m}_u^2)} \right) \\
&- \frac{N_c}{2\pi^2} \kappa^2 \frac{(m_u + m_d)^2}{F_0^2} \int_0^1 dt \{ [2\bar{m}_u^2 - \bar{m}_u \bar{m}_d + 2t(\bar{m}_d^2 - \bar{m}_u^2) - 3t(1-t)q^2] \ln \left(1 - \frac{t(1-t)q^2}{\bar{m}_u^2 + t(\bar{m}_d^2 - \bar{m}_u^2)} \right) \right. \\
&\quad \left. + t(1-t)q^2 \left(2 - \frac{\bar{m}_u \bar{m}_d}{\bar{m}_u^2 + t(\bar{m}_d^2 - \bar{m}_u^2)} \right) \right\}.
\end{aligned} \tag{23}$$

It should be pointed out that $\alpha(q^2; m_u, m_d)$ is order q^2 at least and $\beta(q^2; m_u, m_d)$ is order q^4 at least. Since in this paper we focus on pseudoscalar meson spectrums, these high order derivative terms should obey motion equation of pseudoscalar mesons. In momentum space, the motion equation of physical pseudoscalar mesons is generally written

$$(q^2 - m_\varphi^2)\varphi(q) = -i f_\varphi q^\mu A_\mu^{(\varphi)}(-q), \tag{24}$$

where m_φ and f_φ are physical mass and decay constants of pseudoscalar, e.g., $m_\pi = 135\text{MeV}$ and $f_\pi = 185.2\text{MeV}$. Due to this motion equation, we have

$$\begin{aligned}
\alpha(q^2; m_u, m_d) a_\mu^+(x) \pi^-(q) &= \alpha(m_\pi^2; m_u, m_d) a_\mu^+(x) \pi^-(q), \\
\beta(q^2; m_u, m_d) \pi^+(q) \pi^-(q) &= \beta(m_\pi^2; m_u, m_d) \pi^+(q) \pi^-(q) - \frac{i}{2} q^\mu f_\pi \beta'(m_\pi^2; m_u, m_d) (a_\mu^+(x) \pi^-(q) + c.c.),
\end{aligned} \tag{25}$$

where

$$\beta'(m_\pi^2; m_u, m_d) = \frac{d}{dq^2} \beta(q^2; m_u, m_d)|_{q^2=m_\pi^2}. \tag{26}$$

Thus eq.(22) can be rewritten

$$\begin{aligned}
\mathcal{L}_2[\pi^\pm(x)] &= \frac{F^2(m_u, m_d)}{4} \partial_\mu \pi^+ \partial^\mu \pi^- - \frac{F_0^2}{4} \{ \bar{M}^2(m_u, m_d) + \beta(m_\pi^2; m_u, m_d) \} \pi^+ \pi^- \\
&+ \{ \frac{\bar{f}^2(m_u, m_d)}{2} + \frac{\alpha(m_\pi^2; m_u, m_d)}{2} + \frac{F_0^2}{4F(m_u, m_d)} f_\pi \beta'(m_\pi^2; m_u, m_d) \} (a_\mu^+ \partial^\mu \pi^- + c.c.).
\end{aligned} \tag{27}$$

The two-point effective action concerning to masses and decay constants of neutral pion and η_8 can be evaluated similarly. However the decay constants for neutral mesons cannot be extracted directly from the data. It means that the decay constants for neutral mesons cannot be used to determine light current quark masses. Therefore, in this paper we do not need to evaluate the decay constants for neutral pion and η_8 . The effective action containing the quadratic terms of neutral pion and η_8 is then

$$\begin{aligned}
&iS_2[\pi^0, \eta_8] \\
&= \frac{g_A^2}{8} \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \sum_{j=u,d,s} q^2 P_j(q) P_j(-q) \text{Tr}[\gamma^\mu \gamma_5 S_F^{(j)}(k-q) \gamma^\nu \gamma_5 S_F^{(j)}(k)] \\
&+ \frac{i}{2} g_A \kappa \sum_{j=u,d,s} m_j \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} (i q_\mu) P_j(q) P_j(-q) \text{Tr}[\gamma^\mu \gamma_5 S_F^{(j)}(k-q) \gamma_5 S_F^{(j)}(k)] \\
&- \frac{\kappa^2}{2} \sum_{j=u,d,s} m_j^2 \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} P_j(q) P_j(-q) \text{Tr}[\gamma_5 S_F^{(j)}(k-q) \gamma_5 S_F^{(j)}(k)] \\
&= \frac{N_c}{(4\pi)^2} \Gamma(2 - \frac{D}{2}) g_A^2 i \int \frac{d^4 q}{(2\pi)^4} q^2 \sum_{j=u,d,s} \bar{m}_j^2 q^2 P_j(q) P_j(-q) \int_0^1 dt \left(\frac{\mu^2}{\bar{m}_u^2 - t(1-t)q^2} \right)^{\frac{\epsilon}{2}}
\end{aligned}$$

$$\begin{aligned}
& + \frac{2N_c}{(4\pi)^2} \Gamma(2 - \frac{D}{2}) \kappa g_A i \int \frac{d^4 q}{(2\pi)^4} q^2 \sum_{j=u,d,s} m_j \bar{m}_j q^2 P_j(q) P_j(-q) \int_0^1 dt \left(\frac{\mu^2}{\bar{m}_u^2 - t(1-t)q^2} \right)^{\frac{\epsilon}{2}} \\
& + \frac{2N_c}{(4\pi)^2} \kappa^2 i \int \frac{d^4 q}{(2\pi)^4} \sum_{j=u,d,s} m_j^2 P_j(q) P_j(-q) \int_0^1 dt \left(\frac{\mu^2}{\bar{m}_u^2 - t(1-t)q^2} \right)^{\frac{\epsilon}{2}} \\
& \times \{ \Gamma(1 - \frac{D}{2}) [\bar{m}_j^2 - t(1-t)q^2] + 2t(1-t)q^2 \Gamma(2 - \frac{D}{2}) \}. \tag{28}
\end{aligned}$$

The divergences in the above effective action can be canceled by eqs.(18) and (21). Then from eqs.(19) and (28) we can obtain $O(N_c)$ effective lagrangian describing two-point vertex of π^0 and η_8 as follow

$$\mathcal{L}_2[\pi^0(x), \eta_8(x)] = \sum_{i=u,d,s} \{ \frac{F_i^2(m_i)}{8} \partial_\mu P_i \partial^\mu P_i - \frac{F_0^2}{8} \bar{M}_i^2(m_i) P_i^2 - \frac{F_0^2}{8} \int \frac{d^4 q}{(2\pi)^4} e^{iq \cdot x} \beta_i(q^2; m_i) P_i(q) P_i(x) \}, \tag{29}$$

where

$$\begin{aligned}
F_i^2(m_i) &= \frac{F_0^2}{2} + 3g^2 g_A^2 m_i (2m + m_i) + 6g^2 \kappa g_A m_i \bar{m}_i + 8\kappa^2 \left(\frac{g^2}{4} - \frac{F_0^2 B_0}{48m^3} - \frac{N_c}{48\pi^2} \right) m_i^2 \\
&\quad - \frac{N_c}{2\pi^2} (g_A^2 \bar{m}_i^2 + 2g_A \kappa m_i \bar{m}_i + \kappa^2 m_i^2) \ln x_i^2, \\
\bar{M}_i^2(m_i) &= (B_0 + \frac{N_c}{\pi^2} \frac{m^3}{F_0^2} \ln x_i^2) m_i x_i^2 (x_i - \frac{m_i}{m} \kappa^2), \\
\beta_i(q^2; m_i) &= \frac{N_c}{2\pi^2 F_0^2} q^2 \{ \frac{\kappa^2}{3} m_i^2 + (g_A^2 \bar{m}_i^2 + 2\kappa m_i \bar{m}_i + 2\kappa^2 m_i^2 [\bar{m}_i^2 q^{-2} - 3t(1-t)]) \int_0^1 dt \ln(1 - \frac{t(1-t)q^2}{\bar{m}_i^2}) \}.
\end{aligned} \tag{30}$$

Since auxiliary fields P_i do not lie in physical hadron spectrums, the equation of motion (24) can not be used in (29) simply. In section 5, we will use propagator method to deal with the terms with high power momenta in (29) and diagonalize $\pi_3 - \eta_8$ mixing.

IV. MESON LOOPS AND RENORMALIZATION

Purpose of this section is to evaluate one-loop effects of pseudoscalar mesons. Due to parity conservation, there are only tadpole diagrams of pseudoscalar mesons contributing to masses of decay constants of 0^- mesons (fig. 1). Since ChCQM is a nonrenormalizable effective theory, it is very difficult to calculate meson loop effects completely. However, we can expect that, in mass spectrums and decay constants of pseudoscalar mesons, the dominant one-loop effects is generated by the lowest order effective lagrangian, because neglect is $O(m_q^3)$ and suppressed by N_c^{-1} expansion. Fortunately, the formalism is renormalized up to this order.

The lowest order effective lagrangian is well-known

$$\begin{aligned}
\mathcal{L}_2 &= \frac{F_0^2}{16} < \nabla_\mu U \nabla^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger > \\
&= \frac{F_0^2}{48} < [\lambda^a, \Delta_\mu] [\lambda^a, \Delta^\mu] > + \frac{3F_0^2}{256} < \lambda^a \lambda^a (\xi \chi^\dagger \xi + \xi^\dagger \chi \xi^\dagger) >, \tag{31}
\end{aligned}$$

where $\lambda^a (a = 1, 2, \dots, 8)$ are Gell-Mann matrices, $\chi = 2B_0 \tilde{\chi}$ and the following SU(N) completeness relation have been used

$$\begin{aligned}
\sum_{a=1}^{N^2-1} < \lambda^a A \lambda^a B > &= -\frac{2}{N} < AB > + 2 < A > < B >, \\
\sum_{a=1}^{N^2-1} < \lambda^a A > < \lambda^a B > &= 2 < AB > - \frac{2}{N} < A > < B >. \tag{32}
\end{aligned}$$

To evaluate the one-loop graphs generated by this lagrangian, we consider the quantum fluctuation $\varphi(x) = \varphi^a(x) \lambda^a$ around the solution $\bar{U}(x) = \bar{\xi}^2(x)$ to the classical equations of motion,

$$U = \bar{\xi} e^{i\varphi} \bar{\xi}. \quad (33)$$

Substituting expansion(33) into \mathcal{L}_2 and retaining terms up to and including φ^2 one obtains

$$\mathcal{L}_2 \rightarrow \bar{\mathcal{L}}_2 + \frac{F_0^2}{8}(\partial_\mu \varphi^a \partial^\mu \varphi^a - m_\varphi^2 \varphi^a \varphi^a) - \frac{F_0^2}{16} < [\varphi, \Delta_\mu][\varphi, \Delta^\mu] + \frac{1}{4}\{\varphi, \varphi\}(\bar{\xi}\chi^\dagger \bar{\xi} + \bar{\xi}^\dagger \chi \bar{\xi}^\dagger - 2\mathcal{M}) >, \quad (34)$$

where we have omitted some terms which do not contribute to masses and decay constants via one-loop graphs. The contribution of tadpole graphs can be calculated easily

$$\begin{aligned} \mathcal{L}_2^{(\text{tad})} &= -\frac{1}{4} < [\lambda^a, \Delta_\mu][\lambda^a, \Delta^\mu] + \frac{1}{4}\{\lambda^a, \lambda^a\}(\bar{\xi}\chi^\dagger \bar{\xi} + \bar{\xi}^\dagger \chi \bar{\xi}^\dagger - 2\mathcal{M}) > \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2 - m_\varphi^2 + i\epsilon} \\ &= \frac{1}{4}(m_\varphi^2 \lambda - \frac{m_\varphi^2}{16\pi^2} \ln \frac{m_\varphi^2}{\mu^2}) < [\lambda^a, \Delta_\mu][\lambda^a, \Delta^\mu] + \frac{1}{4}\{\lambda^a, \lambda^a\}(\bar{\xi}\chi^\dagger \bar{\xi} + \bar{\xi}^\dagger \chi \bar{\xi}^\dagger - 2\mathcal{M}) >, \end{aligned} \quad (35)$$

where

$$\lambda = \frac{1}{16\pi^2} \left\{ \frac{2}{\epsilon} + \ln(4\pi) + \gamma + 1 \right\}.$$

Comparing eq.(35) and eq.(31) we can see that the divergence λ can be absorbed by free parameters F_0 and B_0 . Thus the sum of tree graphs and tadpole contribution is

$$\mathcal{L}_2^{(t)} = \frac{F_0^2}{48}(1 - 3\mu_\varphi)v < [\lambda^a, \Delta_\mu][\lambda^a, \Delta^\mu] > + \frac{3F_0^2}{256}(1 - \frac{8}{3}\mu_\varphi) < \lambda^a \lambda^a (\xi \chi^\dagger \xi + \xi^\dagger \chi \xi^\dagger) >, \quad (36)$$

where

$$\mu_\varphi = \frac{m_\varphi^2}{4\pi^2 F_0^2} \ln \frac{m_\varphi^2}{\mu^2}. \quad (37)$$

In lagrangian(36) we appoint that $m_\varphi = m_\pi$ when $a = 1, 2, 3$, $m_\varphi = m_K$ when $a = 4, 5, 6, 7$ and $m_\varphi = m_{\eta_8}$ when $a = 8$.

V. LIGHT QUARK MASS DETERMINATION BEYOND THE CHIRAL PERTURBATION EXPANSION

For extracting $(m_{K^+})_{\text{QCD}}$ from experimental data, the electromagnetic mass splitting of K -meson is required. The prediction of Dashen theorem [23], $(m_{K^+} - m_{K^0})_{e.m.} = 1.3\text{MeV}$, has been corrected in serveral recent analysis with considering contribution from vector meson exchange. A larger correction is first obtained by Donoghue, Holstein and Wyler [24], who find $(m_{K^+} - m_{K^0})_{e.m.} = 2.3\text{MeV}$. Then Bijmens and Prades [26], who evaluated both long-distance contribution using ENJL model and short-distance contribution using perturbative QCD and factorization, find $(m_{K^+} - m_{K^0})_{e.m.} = 2.4 \pm 0.3\text{MeV}$ at $\mu = m_\rho$. Gao *et.al.* [27] also gave $(m_{K^+} - m_{K^0})_{e.m.} = 2.5\text{MeV}$. Baur and Urech [25], however, obtained a smaller correction, $(m_{K^+} - m_{K^0})_{e.m.} = 1.6\text{MeV}$ at $\mu = m_\rho$. In addition, calculation of lattice QCD [28] found $(m_{K^+} - m_{K^0})_{e.m.} = 1.9\text{MeV}$. These estimates indicate that the corrections to Dashen theorem are indeed substantial. In this the present paper, we average the above results and take $m_{K^+} - m_{K^0})_{e.m.} = 2.1 \pm 0.1\text{MeV}$ at energy scale $\mu = m_\rho$.

From eqs.(27) and (36), the masses and decay constants of koan and chrage pion can be obtained via solve the following equations

$$\begin{aligned} m_{\pi^+}^2 &= \frac{F_0^2}{F_R^2(m_u, m_d)} \{ \bar{M}_R^2(m_u, m_d) + \beta(m_\pi^2; m_u, m_d) \} \\ m_{K^+}^2 &= \frac{F_0^2}{F_R^2(m_u, m_s)} \{ \bar{M}_R^2(m_u, m_s) + \beta(m_K^2; m_u, m_s) \}, \\ m_{K^0}^2 &= \frac{F_0^2}{F_R^2(m_d, m_s)} \{ \bar{M}_R^2(m_d, m_s) + \beta(m_K^2; m_d, m_s) \}, \\ f_{\pi^+} &= \frac{\bar{f}_R^2(m_u, m_d)}{F_R(m_u, m_d)} + \frac{\alpha(m_\pi^2; m_u, m_d)}{F_R(m_u, m_d)} + \frac{F_0^2}{2F_R^2(m_u, m_d)} f_{\pi^+} \beta'(m_\pi^2; m_u, m_d), \end{aligned} \quad (38)$$

$$f_{K^+} = \frac{\bar{f}_R^2(m_u, m_s)}{F_R(m_u, m_s)} + \frac{\alpha(m_K^2; m_u, m_s)}{F_R(m_u, m_s)} + \frac{F_0^2}{2F_R^2(m_u, m_s)} f_K \beta'(m_K^2; m_u, m_s),$$

$$f_{K^0} = \frac{\bar{f}_R^2(m_d, m_s)}{F_R(m_d, m_s)} + \frac{\alpha(m_K^2; m_d, m_s)}{F_R(m_d, m_s)} + \frac{F_0^2}{2F_R^2(m_d, m_s)} f_K \beta'(m_K^2; m_d, m_s),$$

where subscript “R” denotes renormalized quantity,

$$\begin{aligned} F_R^2(m_u, m_d) &= F^2(m_u, m_d) - F_0^2(2\mu_\pi + \mu_K), \\ F_R^2(m_i, m_s) &= F^2(m_i, m_s) - \frac{3}{4}F_0^2(\mu_\pi + 2\mu_K + \mu_{\eta_8}), \quad (i = u, d) \\ \bar{M}_R^2(m_u, m_d) &= \bar{M}^2(m_u, m_d) - B_0(m_u + m_d)(\frac{3}{2}\mu_\pi + \mu_K + \frac{1}{6}\mu_{\eta_8}), \\ \bar{M}_R^2(m_i, m_s) &= \bar{M}^2(m_i, m_s) - B_0(m_i + m_s)(\frac{3}{4}\mu_\pi + \frac{3}{2}\mu_K + \frac{5}{12}\mu_{\eta_8}), \\ \bar{f}_R^2(m_u, m_d) &= \bar{f}^2(m_u, m_d) - F_0^2(2\mu_\pi + \mu_K), \\ \bar{f}_R^2(m_i, m_s) &= \bar{f}^2(m_i, m_s) - \frac{3}{4}F_0^2(\mu_\pi + 2\mu_K + \mu_{\eta_8}), \end{aligned} \quad (39)$$

Here the quantity μ_φ is defined in (37) and depend on the renormalization scale μ . It has been recognized that the scale plays an important role in low-energy QCD. In this formalism, many parameters, such as light current quark masses, constituent quark mass m , axial-vector coupling constant g_A , are scale-dependent. The characteristic scale of the model is described by the universal coupling constant g which is determined by the first KSRF sum rule at energy scale $\mu = m_\rho$. Hence we take $\mu = m_\rho = 770\text{MeV}$ in μ_φ , and the physical masses and decay constants, however, should be independent of the renormalization scale.

In order to obtain the masses of π^0 and η^8 , the $\pi_3 - \eta_8$ mixing in eq.(29) must be diagonalized. Eq.(29) together with eq.(36) lead to the quadratic terms for the π_3 and η_8 are of the form

$$S_2[\pi_3, \eta_8] = \int \frac{d^4q}{(2\pi)^4} \left\{ \frac{1}{2}(q^2 - M_3^2(q^2))\pi_3^2 + \frac{1}{2}(q^2 - M_8^2(q^2))\eta_8^2 - M_{38}^2(q^2)\pi_3\eta_8 \right\} \quad (40)$$

where

$$\begin{aligned} M_3^2(q^2) &= \frac{F_0^2}{F_3^2} \{ \bar{M}_u^2(m_u) + \bar{M}_d^2(m_d) + \beta_u(q^2; m_u) + \beta_d(q^2; m_d) - 2B_0\hat{m}(\frac{3}{2}\mu_\pi + \mu_K + \frac{1}{6}\mu_{\eta_8}) \}, \\ M_8^2(q^2) &= \frac{F_0^2}{3F_8^2} \{ \bar{M}_u^2(m_u) + \bar{M}_d^2(m_d) + 4\bar{M}_s^2(m_s) + \beta_u(q^2; m_u) + \beta_d(q^2; m_d) + 4\beta_s(q^2; m_s) \\ &\quad - 2B_0(\hat{m} + 2m_s)(2\mu_K + \frac{2}{3}\mu_{\eta_8}) - 2B_0\hat{m}(-\frac{3}{2}\mu_\pi + \mu_K + \frac{1}{2}\mu_{\eta_8}) \}, \\ M_{38}^2(q^2) &= \frac{F_0^2}{\sqrt{3}F_3F_8} \{ \bar{M}_u^2(m_u) - \bar{M}_d^2(m_d) + \beta_u(q^2; m_u) - \beta_d(q^2; m_d) - B_0(m_u - m_d)(\frac{3}{2}\mu_\pi + \mu_K + \frac{1}{6}\mu_{\eta_8}) \} \\ &\quad + q^2 \frac{F_u^2(m_u) - F_d^2(m_d)}{\sqrt{3}F_3F_8}, \end{aligned} \quad (41)$$

with $\hat{m} = (m_u + m_d)/2$ and

$$\begin{aligned} F_3^2 &= F_u^2(m_u) + F_d^2(m_d) - F_0^2(2\mu_\pi + \mu_K), \\ F_8^2 &= \frac{1}{3} \{ F_u^2(m_u) + F_d^2(m_d) + 4F_s^2(m_s) \} - 3F_0^2\mu_K. \end{aligned} \quad (42)$$

Due to $\pi_3 - \eta_8$ mixing, the “physical” propagators of π^0 and η_8 are obtained via the chain approximation in momentum space

$$\begin{aligned} \frac{i}{q^2 - m_{\pi^0}^2 + i\epsilon} &= \frac{i}{q^2 - M_3^2(q^2) + i\epsilon} + \frac{iM_{38}^4(q^2)}{(q^2 - M_8^2(q^2) + i\epsilon)(q^2 - M_3^2(q^2) + i\epsilon)^2} + \dots \\ &= \frac{i}{q^2 - M_3^2(q^2) - \frac{M_{38}^4(q^2)}{q^2 - M_8^2(q^2)} + i\epsilon}, \end{aligned}$$

$$\begin{aligned}
\frac{i}{q^2 - m_{\eta_8}^2 + i\epsilon} &= \frac{i}{q^2 - M_8^2(q^2) + i\epsilon} + \frac{iM_{38}^4(q^2)}{(q^2 - M_3^2(q^2) + i\epsilon)(q^2 - M_8^2(q^2) + i\epsilon)^2} + \dots \\
&= \frac{i}{q^2 - M_8^2(q^2) - \frac{M_{38}^4(q^2)}{q^2 - M_3^2(q^2)} + i\epsilon}.
\end{aligned} \tag{43}$$

Then the masses of π^0 and η_8 are just solutions of the following equations,

$$\begin{aligned}
m_{\pi^0}^2 &= M_3^2(m_{\pi^0}^2) + \frac{M_{38}^2(m_{\pi^0}^2)}{m_{\pi^0}^2 - M_8^2(m_{\pi^0}^2)}, \\
m_{\eta_8}^2 &= M_8^2(m_{\eta_8}^2) + \frac{M_{38}^2(m_{\eta_8}^2)}{m_{\eta_8}^2 - M_8^2(m_{\eta_8}^2)}.
\end{aligned} \tag{44}$$

In eqs. (23) and (30), the parameters κ , F_0 and B_0 are still not determined. In order to determine them and three light quark masses, six inputs are required. In this paper we choose $f_{\pi^+} = 185.2 \pm 0.5 \text{ MeV}$, $f_{K^+} = 226.0 \pm 2.5 \text{ MeV}$, $m_{\pi^0} = 134.98 \text{ MeV}$, $m_{K^0} = 497.67 \text{ MeV}$ and $(m_{K^+})_{\text{QCD}} = 491.6 \pm 0.1 \text{ MeV}$. Another input is $m_d - m_u = 3.9 \pm 0.22 \text{ MeV}$, which is extracted from $\omega \rightarrow \pi^+ \pi^-$ decay at energy scale $\mu = m_\rho$ in ref. [15]. Recalling $m = 480 \text{ MeV}$, $g_A = 0.75$ and $g = \pi^{-1}$ for $N_c = 3$, we can fit light quark masses as in table 1.

In table 1, the errors of results are from uncertainties in decay constant of K^+ , electromagnetic mass splitting of K -mesons and isospin violation parameter $m_d - m_u$ respectively. The first column in table 1 we show the results for $f_{K^+} = 223.5 \text{ MeV}$. The third column corresponds the center value of f_{K^+} , 226.0 MeV and the fifth column corresponds $f_{K^+} = 228.5 \text{ MeV}$. Then from table 1 we have

$$\begin{aligned}
m_s &= 160 \pm 15.5 \text{ MeV}, & m_d &= 7.9 \pm 2.7 \text{ MeV}, & m_u &= 4.1 \pm 1.5 \text{ MeV}, \\
\frac{m_s}{m_d} &= 20.2 \pm 3.0, & \frac{m_u}{m_d} &= 0.5 \pm 0.09.
\end{aligned} \tag{45}$$

Here the large errors are from the uncertainty of f_{K^+} . From table 1 we also have $(m_{\pi^+} - m_{\pi^0})_{\text{QCD}} = -0.25 \text{ MeV}$ (in this paper the contribution from $\pi_3 - \eta'$ mixing is neglected). This result allows that the electromagnetic mass splitting of pion is $(m_{\pi^+} - m_{\pi^0})_{\text{e.m.}} = 4.8 \text{ MeV}$.

	Fit 1	Fit 2	Fit 3	Fit 4	Fit 5
$\dagger f_{\pi^+}$	185.2	185.2	185.2	185.2	185.2
$\dagger f_{K^+}$	223.5	224.1	226.0	227.8	228.5
$\dagger m_{\pi^0}$	134.98	134.98	134.98	134.98	134.98
$\dagger m_{K^0}$	497.67	497.67	497.67	497.67	497.67
$\dagger (m_{K^+})_{\text{QCD}}$	491.6 ± 0.1	491.6 ± 0.1	491.6 ± 0.1	491.6 ± 0.1	491.6 ± 0.1
$\dagger B(\omega \rightarrow \pi\pi)$	1.95%	1.95%	$(2.11 \pm 0.20)\%$	$(2.21 \pm 0.25)\%$	$(2.21 \pm 0.30)\%$
m_s	144.2	150.0	161.8	172.0	175.4
m_d	6.26	6.83	7.94 ± 0.07	8.94 ± 0.1	9.53 ± 0.11
m_u	2.56	3.13	4.10 ∓ 0.07	5.03 ∓ 0.1	5.63 ∓ 0.11
$m_u + m_d$	8.92	9.96	12.04	13.97	15.16
$m_d - m_u$	3.7	3.7	3.84 ± 0.14	3.9 ± 0.2	3.9 ± 0.22
f_{K^0}	224.0	224.7	226.6	228.4	229.0
$(m_{\pi^+})_{\text{QCD}}$	134.74	134.74	134.73	134.73	134.73
m_{η_8}	608.8	616.9	628.8	636.8	641.0
κ	0.5	0.4	0.3	0.25	0.2
F_0	156.95	156.7	156.1	155.6	155.3
B_0	2141.1	1886.6	1551.1	1328.6	1216.8

TABLE I. Light current quark masses predicted by the masses and decay constants of pseudoscalar mesons at energy scale $\mu = m_\rho$. Here κ is dimensionless, other dimensionful quantities are in MeV, and \dagger denotes input. The formula for branching ratio of $\omega \rightarrow \pi^+ \pi^-$ can be found in ref.[15]. It determines the isospin breaking parameter $m_d - m_u$.

In this paper we study information on light quark masses at energy scale $\mu = m_\rho$ in the framework of chiral constituent quark model. The analysis of quark masses beyond the next to leading order in the chiral expansion is a challenging subject. The attempt is stopped in ChPT due to the difficulties mentioned in Introduction. In approach of quark model, however, it is not necessary to treat the light current quark masses as the quantities used to construct a perturbative expansion. In addition, since the light current quark masses can be defined uniquely, the Kaplan-Manohar ambiguity is avoided in this formalism. From eq. (45) $m_u \neq 0$ is also confirmed in our results. This conclusion supports viewpoint of ref. [18,8] but disagree with one of ref. [17].

Our results are yielded by a non-perturbative method and contain complete information on light quark masses. Not only quark mass ratios but also individual light quark masses are obtained. The results agree with one obtained by other approaches(e.g., QCD sum rule or ChPT) well.

Finally, it is interesting to expand non-perturbation results (38) up to the next to leading order of the chiral expansion and compare with one of ChPT. If we neglect the mass difference $m_d - m_u$, up to this order the masses of decay constants of kaon and pion read

$$\begin{aligned}
m_\pi^2 &= B_0(m_u + m_d)\{1 + \frac{1}{2}\mu_\pi - \frac{1}{6}\mu_{\eta_8} + [\frac{1}{2}(3 - \kappa^2) + \frac{3m^2}{\pi^2 F_0^2}(\frac{m}{B_0} - \kappa g_A - \frac{g_A^2}{2} - \frac{B_0}{6m}g_A^2)]\frac{m_u + m_d}{m}\}, \\
m_K^2 &= B_0(\hat{m} + m_s)\{1 + \frac{1}{3}\mu_{\eta_8} + [\frac{1}{2}(3 - \kappa^2) + \frac{3m^2}{\pi^2 F_0^2}(\frac{m}{B_0} - \kappa g_A - \frac{g_A^2}{2} - \frac{B_0}{6m}g_A^2)]\frac{\hat{m} + m_s}{m}\}, \\
f_\pi &= F_0\{1 - \mu_\pi - \frac{1}{2}\mu_K + \frac{3}{2\pi^2}g_A^2\frac{m(m_u + m_d)}{F_0^2}\}, \\
f_K &= F_0\{1 - \frac{3}{8}\mu_\pi - \frac{3}{4}\mu_K - \frac{3}{8}\mu_{\eta_8} + \frac{3}{2\pi^2}g_A^2\frac{m(\hat{m} + m_s)}{F_0^2}\}.
\end{aligned} \tag{46}$$

Comparing the above equation with one of ChPT [3], we predict the $O(p^4)$ chiral coupling constants, L_4 , L_5 , L_6 and L_8 , as follow

$$\begin{aligned}
L_4 &= L_6 = 0, & L_5 &= \frac{3m}{32\pi^2 B_0}g_A^2, \\
L_8 &= \frac{F_0^2}{128B_0m}(3 - \kappa^2) + \frac{3m}{64\pi^2 B_0}(\frac{m}{B_0} - \kappa g_A - \frac{g_A^2}{2} - \frac{B_0}{6m}g_A^2) + \frac{L_5}{2}.
\end{aligned} \tag{47}$$

Numerically, inuptting $\kappa = 0.35 \pm 0.15$, $F_0 = 0.156\text{GeV}$ and $B_0 = 1.6 \pm 0.4\text{GeV}$, we obtain $L_5 = (1.6 \pm 0.3) \times 10^{-3}$ and $L_8 = (0.7 \pm 0.5) \times 10^{-3}$. These value well agree with one of ChPT, $L_5 = (1.4 \pm 0.5) \times 10^{-3}$ and $L_8 = (0.9 \pm 0.3) \times 10^{-3}$ at energy scale $\mu = m_\rho$. This fact can interpret why the light quark mass ratio in eq. (45) are close to results extracted from ChPT by Leutwyler [9]. The above results indicate that the contribution from scalar meson resonance exchange is small. In fact, in hardon spectrum, there is no scalar meson octet or singlet which belong to composited fields of $q\bar{q}$. Thus it is a *ad hoc* assumption to argue some low energy coupling constants, such as L_5 and L_8 , receiving large contribution from scalar meson exchange.

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